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Viscous resuspension in a bidensity suspension

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Abstract

Experiments were performed in a narrow gap Couette device to study the viscous resuspension process in a bidensity suspension consisting of two types of spherical particles, both having the same size, one of which was heavy, while the other had the same density as that of the suspending fluid. The resuspension height of an initially settled bed of heavy particles was measured over a ten-fold range of applied shear rates, using an imaging technique which was developed to study the particle migration process in concentrate bidisperse suspensions. It was found that, at a given shear rate, the resuspension height of the heavy particles increased with an increase in the concentration of the neutrally buoyant spheres. The experimental results were then compared with the predictions of a bidensity model based on the particle migration theory of previous work, and good agreement was found between the two. © 1999 Elsevier Science Ltd. All rights reserved.

Keywords: Resuspension; Suspensions; Bidensity

1. Introduction

Viscous resuspension is a process whereby, in the presence of shear, an initially settled layer of heavy particles becomes entrained in the bulk fluid, even under conditions of vanishingly small Reynolds numbers. This phenomenon was studied both experimentally and theoretically by, among others, Gadala-Maria (1979), Leighton and Acrivos (1986), Schaflinger et al. (1990), Chapman and Leighton (1991), and more recently by Acrivos et al. (1993). Acrivos et al. (1993) investigated this process in a narrow gap Couette device, where the shear rate was (approximately) constant across the gap and, following Leighton and Acrivos (1986), modelled it by balancing the downward flux of particles due to gravity with an upward flux arising from shear-induced particle diffusion in a direction normal to the plane of shear, from regions of

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high particle concentrations to low. According to this model, the change Δh in the height of the bed, relative to its initial settled value h_o, is a function of the single parameter:

$$A = \frac{9}{2} \frac{\mu_f \gamma}{g h_o \Delta\rho},$$

with γ being the shear rate, μ the viscosity of the pure liquid and $g\Delta\rho$ the buoyancy force, which reflects the ratio of viscous to gravitational forces. Acrivos et al. (1993) found good agreement between the measured resuspension heights and those computed from the solution of their model equation.

In the present paper, we report the results of an experimental investigation of the resuspension process in a narrow gap Couette device where the suspension consisted of two types of spherical particles, both having the same size, one of which was heavy, while the other had the same density as that of the fluid. Unlike the case treated in Acrivos et al. (1993), where a clear fluid layer existed above an initially settled bed, here, the region was occupied initially by a suspension of neutrally buoyant particles of given volume fraction ϕ^*. The migration of particles and the resuspension height were observed using an imaging technique, which was developed to study the shear-induced particle migration in concentrated bidisperse suspensions undergoing shear. It was found that, at the same shear rate, the resuspension height of the heavy particles was enhanced when ϕ^*, the initial concentration of the neutrally buoyant particles, was increased. This observed enhancement will be explained in terms of a bi density model based on the particle migration theory of Leighton and Acrivos (1987b). The experimentally determined resuspension heights were found to be in good agreement with those predicted theoretically.

In the next section, we shall describe our experimental technique and results, and in Section 3 we shall present a simple theoretical model for shear-induced migration in a bidensity suspension (particles having same size but of different density) to explain the enhanced resuspension phenomenon observed in our experiments. The final section is devoted to conclusions.

2. Experimental technique and results

2.1. Apparatus and materials

The experiments were performed in a narrow gap cylindrical Couette device, shown in Fig. 1, which consisted of two cylinders made of high quality plexiglass. The inner radius of the outer stationary cylinder (R_o) was 8.224 cm and the outer radius of the inner rotating cylinder (R_i) was 7.542 cm, giving a gap size equal to 0.682 cm. The inner cylinder was mounted on a shaft which, in turn, was mounted on a motor (ID Corp., California). The bottom of the apparatus was sealed with mercury. This prevented the particles from migrating out of the gap and also created a stress-free boundary. Since the gap-to-inner radius ratio was small, the shear was taken as being constant across the gap.

The particles used in the experiments consisted of acrylic and polystyrene spheres. The former were a lot of class B728 obtained from Zenica Resins, while the latter, obtained from

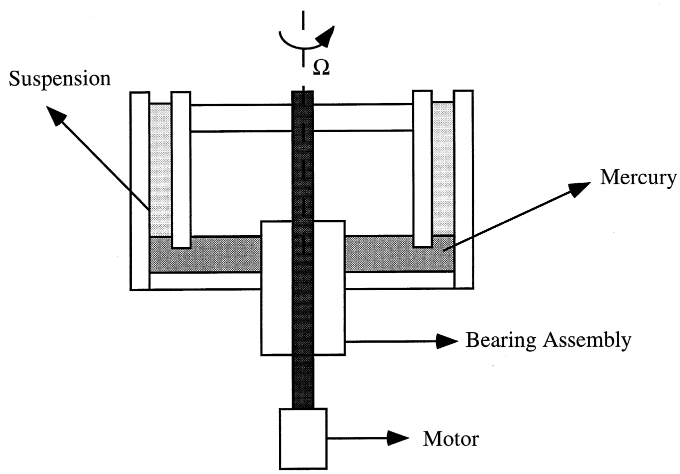

Fig. 1. Schematic of the resuspension in a Couette gap and the experimental apparatus.

Maxiblast Inc., were a lot of class PB-2.5. The density of the acrylic spheres was measured to be 1.165 g/cc and those spheres containing air bubbles were segregated. Similarly, the polystyrene spheres of density 1.045 g/cc were segregated from those containing air bubbles. The polydisperse material was sieved to obtain a uniform fraction in the size range 300–350 μm, with a standard deviation of 25 μm. The acrylic spheres were dyed with black RIT liquid fabric dye and the color coating on the particles was found to be insoluble in the suspending fluid. The latter consisted of a mixture of 40.9% (by volume) Dow Corning 556 and 59.1% Dow Corning 705 fluid which matched the density of the polystyrene particles. The refractive index of the suspending fluid was close to that of polystyrene particles though they were not matched exactly. The pure suspending fluid was Newtonian and had a viscosity of 57.2 cp at the operating temperature of 24 °C.

2.2. Experimental procedure

After sealing the gap with mercury, a sufficient amount (57 ml) of suspending fluid was loaded to a height of 2 cm over the mercury in the Couette gap. The heavy (black) particles were subsequently introduced and the suspension was allowed to rest for three days. The settled height h_0 of the suspension was then measured to be 0.43 and 0.49 cm for two different sets of experimental runs. Hence, the height of the suspending fluid occupying the region above the settled layer was sufficiently large for the purpose of maintaining the clear fluid layer above the resuspended region at all applied shear rates. Next, the suspension was sheared at shear rates varying from 1.48 to 15 s^{-1} which were lower by at least a factor of 20 than the critical shear rate for the onset of Taylor instability in a narrow gap Couette flow, with the fluids used in this study. The change in the equilibrium height of the resuspended layer was measured using the technique described in the next section, and consecutive runs were taken after a sufficient time had elapsed, in order to ensure that steady state had been reached. This procedure was then repeated after adding density matched polystyrene particles (white) to a liquid above the settled bed, and the resulting suspension was sheared for a day to drive out any air bubbles and eliminate any local concentration inhomogeneities present initially as a result of the loading procedure. The experiments were performed for values of the density matched particle concentration ϕ^* equal to 10, 20, 30, 40 and 45%.

2.3. Measurement technique

The migration of the particles, the resuspended height and the particle concentration were observed by viewing the suspension through a plexiglass window using a high resolution (1000×1000 K) CCD camera (Kodak MegaPlus ES1.0) with Infinity Optics (98X total magnification on the monitor), plus a Dantec light sheeting probe that provided an excellent contrast in the video. A three-dimensional traversing system was employed to mount the camera. This allowed us to measure the resuspended height and the concentration at various locations, by traversing the magnified image on the video. Images from the CCD camera were passed via an 8-bit digital video signal to a dedicated image and processing board (Oculus F/64 by Coreco Inc.), which operated on a personal computer equipped with a 200 MHz Intel Pentium processor.

The concentration of the heavy (black) particles was measured by counting of the number of particles in a known volume element $\Delta x \times \Delta y \times \Delta z$, of the flowing suspension. The cross-sectional area, $\Delta x \times \Delta y$, was calculated using the known magnification of the camera optics and the pixel size of the CCD chip and, in our experiments, was found to be 0.5×0.51 mm. The depth of field, Δz, was determined by viewing a 45° inclined plane through the camera with the preset optics, as shown in Fig. 2. The inclined plane had four sets of target columns containing different line pairs per millimeter (1p/mm). The resolution appropriate for the given magnification was then chosen and the distance corresponding to the point at which the resolution disappeared was measured. This distance was read from either the computer monitor or the scale on the inclined plane. Note that there was a factor of $\sqrt{2}$ on the appropriate inclined scale to account for the viewing angle. Using this technique, we measured the depth of view and the range of grey level counts at the camera resolution. In our experiments, the depth of field was found to be 425 ± 25 μm. Next, the number of heavy (black) particles in this volume element was found by using a particle counting procedure in the imaging software (Visilog 5.0). This procedure involves morphology, binarization, threshold, holefill and edge-detection operations. Moreover, the grey level range estimated in the depth of field measurements successfully eliminated the "out of plane" blurred particles during these software manipulations.

The accuracy of the particle counting scheme depends on the quality of the image and the degree of particle overlap. In concentrated suspensions, the high degree of particle overlap makes is difficult to use the counting procedures for concentration estimations. To be sure, the concentration could be measured, in principle, by coloring a known fraction of refractive-index matched particles, and assuming that the concentration of the suspension at any location is proportional to the fraction of the colored particles at that location. In our experimental system, however, it is not possible to match the refractive-indices for bidensity suspensions which have materials with different refractive-indices. Therefore, the concentration profiles of the neutrally buoyant and black particles were not measured throughout the suspension.

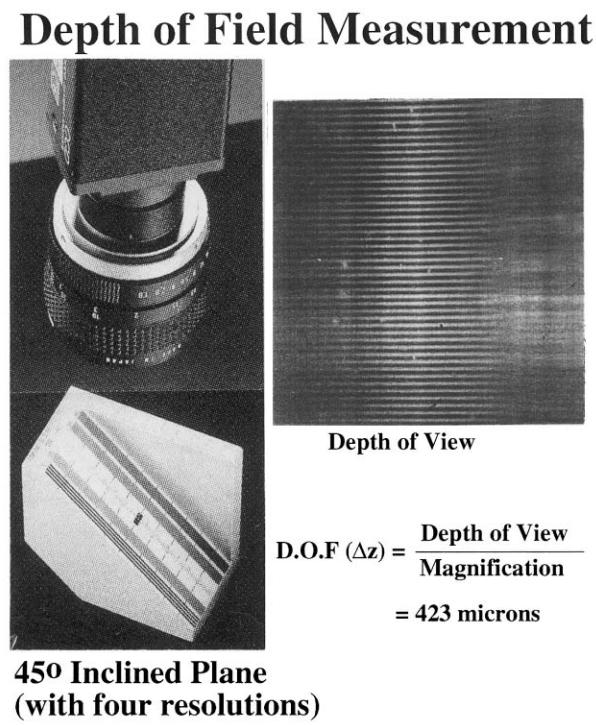

Fig. 2. Schematic diagram of the arrangement of the camera in depth of field measurement.

Instead, in order to ascertain the height of the resuspension, we measured the concentration of the heavy (black) particles only in those regions where it fell below 5–6%. The center of the measurement volume was located at a distance of one particle diameter inside the outer wall of the Couette device using the traversing system. At each location, around 200 images were digitized and processed for the concentration evolution. Subsequently, the concentration of the heavy (black) particles was obtained by ensemble averaging these images.

2.4. Experimental observations and results

Many interesting features were observed during our experiments investigating the shear-induced particle migration in a bidensity suspension. We first measured the resuspension height of the black particles in the absence of the neutrally buoyant particles (mono-density case) for shear rates varying from 1.4–15 s^{-1}. The results were then compared with the theoretical and experimental study of Acrivos et al. (1993) and were found to be in good agreement. The neutrally buoyant particles were then loaded at various concentrations following the procedure described in the previous section. We were then surprised to watch the heavy particles migrating (against gravity!) into the region occupied by the neutrally buoyant particles, and also the neutrally buoyant particles migrating into the region of high concentration of the heavy particles. We observed that, as the initial concentration ϕ^* of the neutrally buoyant particles was increased, an increasing number of heavy (black) particles were entrained into the region containing the neutrally buoyant suspension. Shown in Fig. 3 are the photographs of different experimental runs. Further, unlike the case of the resuspension of heavy particles in clear fluid (the mono-density resuspension process), where a sharp interface was formed

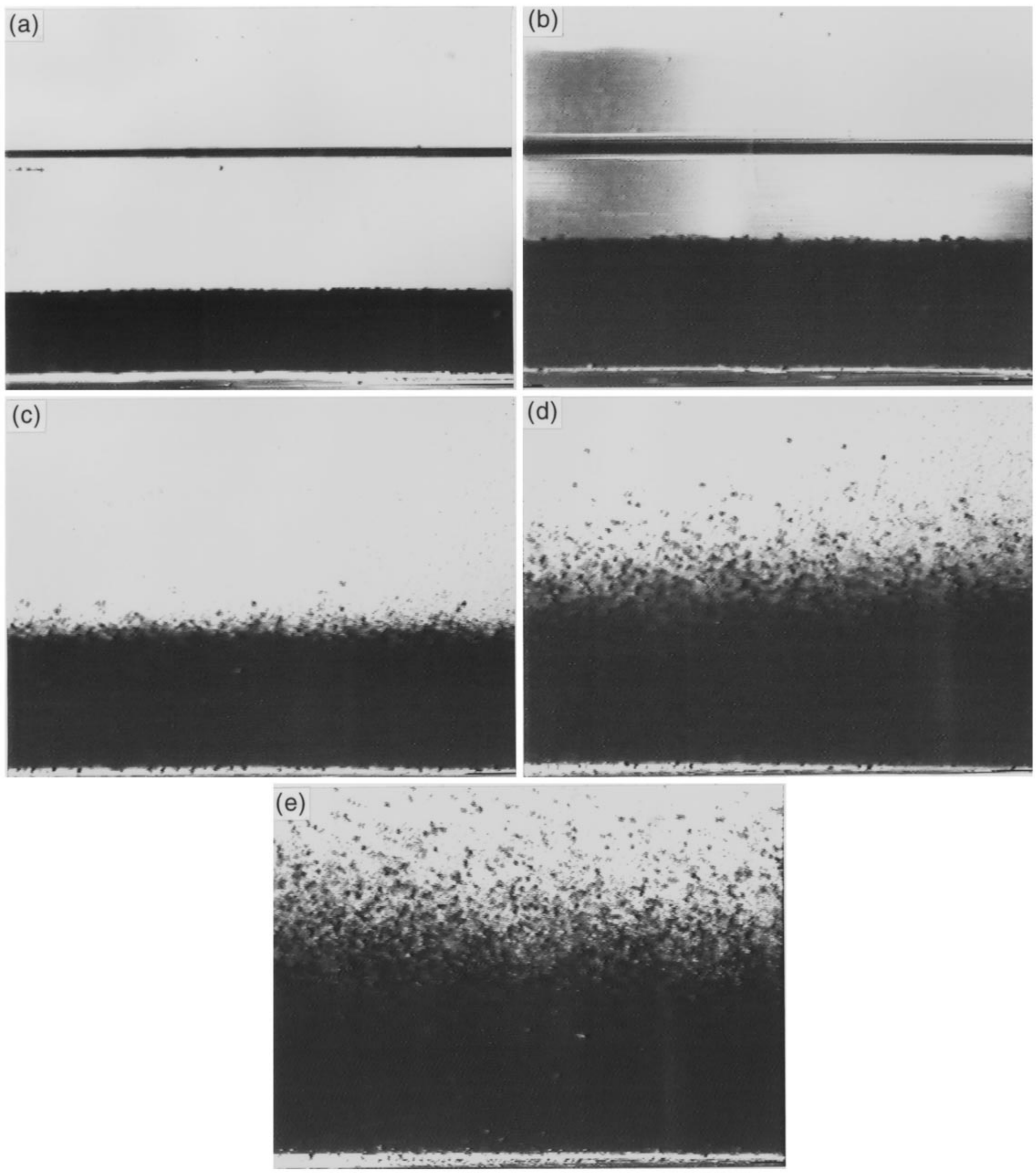

Fig. 3. The photographs of different experimental runs. (a) $A = 0$ and $\phi^* = 0$; (b) $A = 0.5$ and $\phi^* = 0$; (c) $A = 0.5$ and $\phi^* = 0.3$; (d) $A = 0.5$ and $\phi^* = 0.4$; and (e) $A = 0.5$ and $\phi^* = 0.45$.

between the black particle resuspended layer and the fluid above it, here, the concentration of the black particles dropped to a negligible value over an extended region whose width increased with ϕ^*. Hence, it was impossible to locate the position where the concentration of the resuspended black particles was zero. We, therefore, chose to denote the resuspension height of the layer of black particles as that location where the measured concentration of the black particles was approximately 0.02.

This resuspension height was measured using the imaging technique described in the previous section. By making several measurements and then averaging, we found that, in general, this height could be located accurately with a standard deviation of 0.1 mm whenever either the applied shear rate or the concentration of the neutrally buoyant particles was small. In these cases, the concentration of the black particles fell off rapidly to a negligibly small value. Thus, a clear contrast was created at the location where the black particle concentration became small. However, when the concentration of the neutrally buoyant particles was increased, we observed that the number density of the black particles decrease gradually, rather than abruptly, towards the top of the neutrally buoyant layer with the result that, owing to the small gradient in concentration, it was difficult to locate accurately the location at which the concentration of the black particles equalled 2%. We observed that, in these cases, the resuspended height (as defined above) could be located with an error bar shown in Fig. 4.

In the following section, we shall develop a simple theoretical model for shear-induced migration in a bidensity suspension (particles having same size but different densities) to explain our experimental findings and to determine the concentration profile and resuspension height of the black particles.

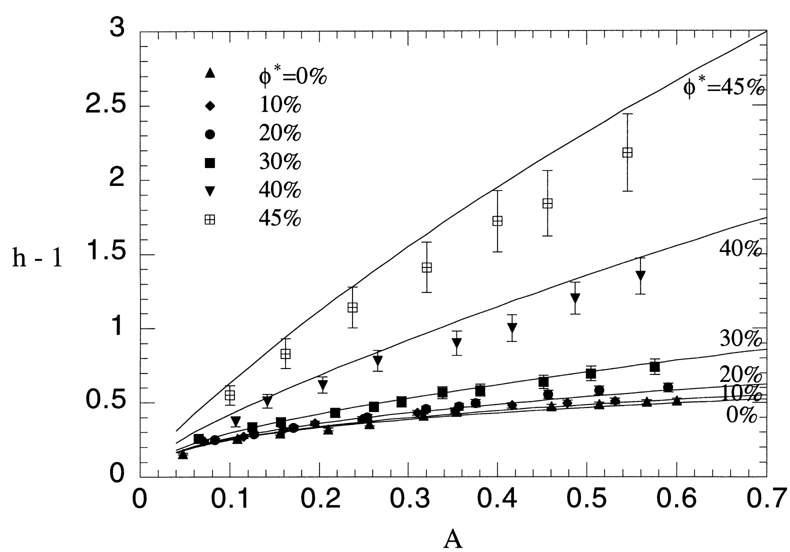

Fig. 4. A comparison between the experimental data for h as a function of A and the theoretical model predictions.

3. Theory

3.1. Shear-induced resuspension in monodisperse suspensions

Consider an initially settled bed of heavy, non-Brownian monodisperse particles immersed in an infinite layer of a suspending fluid in a narrow gap Couette device. In the presence of an externally imposed shear flow, the settled layer is entrained into the bulk fluid and acquires a non-uniform particle concentration profile even under conditions of vanishingly small Reynolds numbers. According to Leighton and Acrivos (1986), the particle concentration profile which is attained results from the combined effect of two particle fluxes: (a) a gravitational flux given by:

$$\frac{2a^2 g\Delta\rho}{9\mu_{\mathrm{f}}}\phi f(\phi), \tag{1}$$

where a is the particle radius, $f(\phi)$ is the hindrance function, μ_{f} is the viscosity of the pure suspending fluid, and $g\Delta\rho$ is the buoyancy force; and (b) a shear-induced flux proportional to particle concentration gradient $\mathrm{d}\phi/\mathrm{d}y$. In turn, the latter consists of two terms: a convective flux ϕu, where u is the particle drift velocity in the absence of gravity due to the presence of a particle concentration gradient, and ϕ is the particle concentration, and a diffusive flux equal to $-\gamma a^2 D_{\mathrm{tr}}(\phi)(\mathrm{d}\phi/\mathrm{d}y)$, where $D_{\mathrm{tr}}(\phi)$ is the particle *tracer* diffusivity rendered dimensionless with γa^2. In addition, when the shear rate γ is constant across the gap, the drift velocity is given by:

$$u = -\gamma a^2 \frac{D_{\mathrm{g}}(\phi)}{\phi}\frac{\mathrm{d}\phi}{\mathrm{d}y}, \tag{2}$$

where $D_{\mathrm{g}}(\phi)$, equal to the gradient diffusivity rendered dimensionless with γa^2, is a function of ϕ only. It should be noted that the shear-induced gradient diffusivity is quite different from the shear-induced coefficient of self- or tracer-diffusion. Shear-induced self diffusion, investigated experimentally by Leighton and Acrivos (1987a) and Eckstein et al. (1977), and via numerical simulation by Bossis and Brady (1987), arises from the random motion of the particles, which occurs as they tumble over one another in a shear flow. Thus, self-diffusion governs the mixing of labelled particles in a sheared suspension of uniform concentration. In contrast, the gradient diffusivity is defined as the ratio of the particle convective flux resulting from a concentration gradient to the magnitude of the gradient.

We use the model proposed by Leighton and Acrivos (1986, 1987a) to obtain:

$$D_{\mathrm{g}}(\phi) = \frac{1}{3}\phi^2\left(1 + \frac{1}{2}e^{8.8\phi}\right) - D_{\mathrm{tr}}(\phi), \tag{3}$$

$$D_{\mathrm{tr}}(\phi) = \frac{1}{2}\phi^2(1 + 0.09e^{7\phi}). \tag{4}$$

We note parenthetically that alternate expressions for $D_{\mathrm{g}}(\phi)$ and $D_{\mathrm{tr}}(\phi)$ have been proposed by Phillips et al. (1992), but the results to be presented below are not significantly affected if one set is chosen over the other.

At steady state, the net particle flux in a sheared suspension vanishes, thus we obtain the flux balance equation:

$$A\{D_g(\phi)+D_{tr}\}\frac{d\phi}{d\xi}=-\phi f(\phi), \tag{5}$$

where $\xi = y/h_o$ and A is defined as:

$$A=\frac{9}{2}\frac{\mu_f\gamma}{g\Delta\rho h_o}. \tag{6}$$

Here, h_o is the initial settled height of the heavy particles in the absence of shear. Therefore, the concentration distribution and the resuspended height of a sheared suspension is obtained by solving (5) subject to the conditions:

$$\begin{aligned}\int_0^h \phi\, d\xi &= \phi_{max},\\ \phi(\xi=h) &= 0,\end{aligned} \tag{7}$$

where h is the resuspension height divided by h_o, and ϕ_{max} is the particle concentration in the settled suspension. Also, as in Leighton and Acrivos (1986), we approximate $f(\phi)$ by means of:

$$f(\phi)=\frac{1-\phi}{\mu_r}, \tag{8}$$

where μ_r is the relative viscosity of the suspension given as:

$$\mu_r=\frac{\mu(\phi)}{\mu_f}=\left[1+\frac{1.5\phi}{1-\dfrac{\phi}{\phi_{max}}}\right]^2, \tag{9}$$

with ϕ_{max} taken here as equal to 0.58. A plot of $h-1$ as function of A (as obtained from the numerical solution of the system given above) is shown in Fig. 4. Hence, provided that an infinite clear fluid layer exists above the settled layer, the resuspension height is predicted to increase monotonically with the strength of the applied shear rate. Previous studies (Leighton and Acrivos 1986; Acrivos et al. 1993) have shown a good agreement between the model predictions and the experiments.

In the following subsection, we shall develop a model for determining the concentration profile and resuspension height when an infinite clear fluid layer above an initially settled bed is replaced by a suspension of neutrally buoyant particles.

3.2. Shear-induced resuspension in a bidensity suspension

Consider an infinite layer of a suspension of neutrally buoyant spherical particles (white) of known concentration ϕ^* above an initially settled bed of heavy spherical particles (black colored) in a narrow gap Couette device. The black spheres (density ρ_1) and the white spheres (density $\rho_2 = \rho_f$) have the same size (radius a). In the presence of an applied shear, the black particles will again resuspend into the white suspension but the diffusion and sedimentation fluxes experienced by these particles will now differ from those given in the previous section. Recently, Revoy and Higdon (1992) have described the results of numerical simulations for the polydisperse sedimentation of equal-sized spheres, e.g. particles of different density in the absence of an imposed shear flow. Using Stokesian dynamics, these authors have shown that the settling velocities of individual particle species can be expressed in terms of two scalar functions of the total volume fraction ϕ, the self-mobility M_0 and the interaction mobility M_1. By rearranging (14) of Revoy and Higdon, we therefore obtain the sedimentation flux, N_{g_i} of the two species as:

$$N_{g_i} = \frac{2}{9}\frac{a^2 g \Delta\rho}{\mu_f}\phi_i\left\{\tilde{M}_o(\phi)\delta_{i1} + \frac{\tilde{M}_1(\phi)}{\phi}\phi_1\right\}, \tag{10}$$

where $\phi_i (i = 1, 2)$ is the concentration of species i and $6\pi\mu_f a M_0 \equiv \tilde{M}_0(\phi)$ (similarly for $\tilde{M}_1(\phi)$, c.f. Figs. 1 and 2, Revoy and Higdon). These two scalar functions can be represented as:

$$\tilde{M}_o(\phi) \sim 1 - \frac{\phi}{\phi_{max}} \tag{11}$$

$$\tilde{M}_o(\phi) + \tilde{M}_1(\phi) \sim f(\phi), \tag{12}$$

where $f(\phi)$ is given by (8).

As mentioned in Section 3.1, two additional fluxes arise in a sheared suspension due to the particle drift velocity and the self or tracer diffusivity. According to the model of Leighton and Acrivos, the presence of gradients in concentration lead to gradients in viscosity which, in turn, cause particles undergoing measurable interactions to be displaced from regions of high viscosity to low, and hence, from regions of high concentration to low. Therefore, in bidensity suspensions of two sets of particles having the same size, the particle drift velocity is proportional to the gradient of the total particle concentration, i.e. $d\phi/d\xi$. Consequently, the convective flux, of particles of species i, becomes

$$\frac{\gamma a^2}{h_o}\phi_i\frac{D_g(\phi)}{\phi}\frac{d\phi}{d\xi}, \tag{13}$$

where $D_g(\phi)$ is given by (3).

The second source of particle migration in a bidensity suspension arises from the random self-diffusion process. In order to explain this, let us consider a case where a suspension of *neutrally buoyant* spheres of the same size is loaded in a Couette gap. Let us further imagine that the particles in the bottom half of the suspension are colored black while the particles in the top half are white. When the suspension is subjected to shear, the black particles interact with the other (black and white) particles in the suspension and suffer random displacement normal to their undisturbed streamlines. As a result of this, some of the black particles will spend part of their time in the white layer and a part in the black layer. Note that here the drift mechanism is absent, owing to the absence of actual gradients in the total concentration of particles. In contrast, the random walk motion leading to the tracer diffusivity requires particles to exchange positions and the tracer diffusivity here is proportional to the total concentration of the black and white particles. The shear-induced diffusive flux, of particles of species i, is then given by:

$$\frac{\gamma a^2}{h_\mathrm{o}} D_\mathrm{tr}(\phi)\frac{\mathrm{d}\phi_i}{\mathrm{d}\xi}, \tag{14}$$

where $D_\mathrm{tr}(\phi)$ is given by (4), since the two sets of particles have the same size. Hence, the shear-induced flux, J_{D_i} of particles of species i is given by:

$$J_{\mathrm{D}_i} = -\frac{\gamma a^2}{h_o}\left\{\phi_i \frac{D_\mathrm{g}(\phi)}{\phi}\frac{\mathrm{d}\phi}{\mathrm{d}\xi} + D_\mathrm{tr}(\phi)\frac{\mathrm{d}\phi_i}{\mathrm{d}\xi}\right\}. \tag{15}$$

Equation (15) is similar in form to an expression for the flux of each species proposed by Shauly et al. (1997), who also proposed a set of model equations for the more general case of polydisperse suspensions containing particles of different sizes.

At steady state, the net flux of the particles of each species in a sheared suspension vanishes, thus the flux balance becomes:

$$A\left\{\phi_i \frac{D_\mathrm{g}(\phi)}{\phi}\frac{\mathrm{d}\phi}{\mathrm{d}\xi} + D_\mathrm{tr}(\phi)\frac{\mathrm{d}\phi_i}{\mathrm{d}\xi}\right\} = -\phi_i\left\{\tilde{M}_\mathrm{o}(\phi)\delta_{i1} + \frac{\tilde{M}_1(\phi)}{\phi}\phi_1\right\}. \tag{16}$$

It is important to note that, within the infinite suspension layer of white particles, the tracer diffusivity of the black particles is now finite rather than being zero, as within the clear fluid layer in the monodisperse case. This makes it possible for the black particles to migrate into the infinite layer of neutrally buoyant particles. In addition, the settling velocity of the black particles is now decreased owing to the presence of the white particles, and, therefore, the magnitude of the hindrance function f is reduced. Similarly, the white particles migrate inside the black resuspended layer. Hence, we need to solve (16) subject to the conditions:

$$\int_0^\infty \phi_1 \mathrm{d}\xi = \phi_\mathrm{max}, \tag{17}$$

which expresses the conservation of the total volume of black particles, and

$$\begin{aligned}&\phi_1 \to 0 \text{ as } \xi \to \infty,\\ &\phi_2 \to \phi^* \text{ as } \xi \to \infty.\end{aligned} \tag{18}$$

Next, we add the two differential equations given by (16) to obtain:

$$\frac{\mathrm{d}\phi}{\mathrm{d}\xi} = -\frac{\{\tilde{M}_0(\phi) + \tilde{M}_1(\phi)\}\phi_1}{A\{D_\mathrm{g}(\phi) + D_\mathrm{tr}(\phi)\}}, \tag{19}$$

which, when integrated using (17) and (18) yields:

$$\int_{\phi*}^{\phi(0)} \frac{\{D_\mathrm{g}(\phi) + D_\mathrm{tr}(\phi)\}}{\{\tilde{M}_0(\phi) + \tilde{M}_1(\phi)\}}\mathrm{d}\phi = \frac{\phi_\mathrm{max}}{A}, \tag{20}$$

where $\phi(0)$, determined by the above expression, is the total concentration at $\xi = 0$. We then multiply (16) by ϕ_2 and by ϕ_1 and subtract to obtain:

$$\frac{dX}{d\xi} = -\frac{X(1-X)\tilde{M}_0(\phi)}{AD_{tr}(\phi)}, \tag{21}$$

where $X = \phi_1/\phi$. On making use of (17)–(19) and integrating (21), we then arrive at:

$$X(0) = 1 - \exp\left[-\int_{\phi*}^{\phi(0)} \frac{\tilde{M}_o(\phi)\{D_g(\phi) + D_{tr}(\phi)\}}{\phi D_{tr}(\phi)\{\tilde{M}_0(\phi) + \tilde{M}_1(\phi)\}} d\phi\right], \tag{22}$$

where $X(0)$, determined by the above, is the fraction of the heavy particles at $\xi = 0$. We then solve (19) and (21) using the Runge–Kutta fourth-order method by marching from $\xi = 0$.

Although, in principle, the concentration of the black particles in the bidensity suspension vanishes only at infinity, we chose, as remarked earlier, to estimate the resuspension height as that value of ξ, where the concentration of black particles ϕ_1 equals 2%. Note that this definition of the resuspension height is consistent with that of our experimental work. Shown in Fig. 4 is the comparison between the experimental data for h as a function of A, and the results of the theoretical model as obtained from the numerical evaluation of (5)–(7) and (18)–(22). The two sets appear to be in good agreement. Figs. 5–7 are the theoretical concentration profiles for different values of A and ϕ^*. It is interesting to note that the lighter white particles are also predicted to migrate, down to $\xi = 0$, for high values of A and ϕ^*. The concentration profiles, shown in Figs. 5–7, are consistent with those observed experimentally (shown in Fig. 3).

4. Conclusions

In this paper, we have reported the results of a theoretical and experimental investigation of a viscous resuspension process in a bidensity suspension sheared in a narrow gap Couette

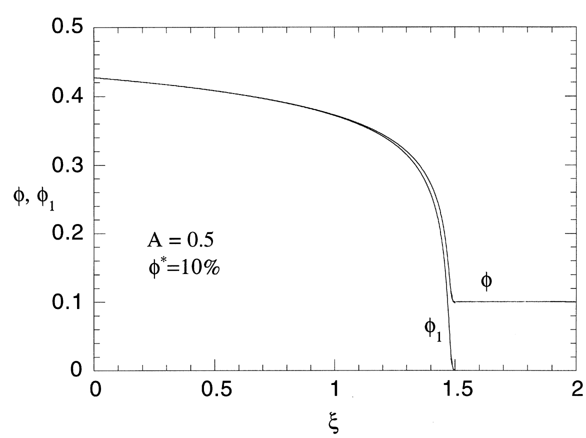

Fig. 5. The theoretical concentration profile for $A = 0.5$ and $\phi^* = 0.1$.

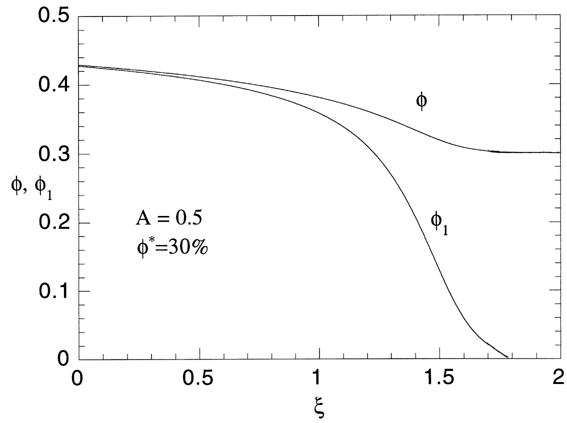

Fig. 6. The theoretical concentration profile for $A = 0.5$ and $\phi^* = 0.3$.

device. This suspension consisted of two types of equi-sized spherical particles, of which one was heavy, while the other had the same density as that of the suspending fluid. Over a ten-fold range of applied shear rates, the resuspension height of the initially settled bed of heavy particles was measured using an imaging technique. The resuspension height of the heavy particles was observed to increase with the concentration of the neutrally buoyant particles. The experimental results were found to be in good agreement with the predictions of a bidensity model based on the Leighton and Acrivos migration theory. The success of the bidensity model has provided strong evidence to the effect that the observed enhancement of the resuspended height is due to the existence of a shear-induced particle tracer diffusivity as well as to the decrease in the settling velocity of the heavy particles due to the presence of the neutrally buoyant particles in the region above the initially settled bed of heavy particles.

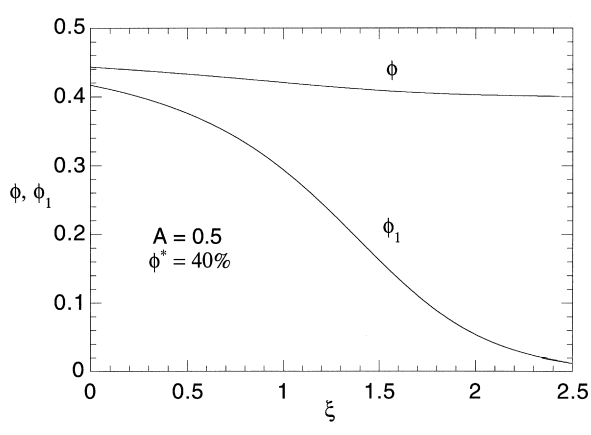

Fig. 7. The theoretical concentration profile for $A = 0.5$ and $\phi^* = 0.4$.

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